Problem

When the units and tens digit of a 3-digit multiple of ten are interchanged, the number obtained becomes lesser than the original number by one-fortieth of the original number. What is the original number?

Solution

3-digit multiple of 10 $\Rightarrow$ Digit in units place is 0 & Digit in hundreds place can't be 0

Let the digit in hundreds place be x and that in tens place be y as shown below:

H	T	U
x	y	0

where x and y are whole numbers such that $1 \leq x \leq 9$ and $0 \leq y \leq 9$

Value of the number $= x \times 100 + y \times 10 + 0 \times 1 = 100x + 10y$

When the units and tens digit are interchanged, number obtained is as shown below:

H	T	U
x	0	y

and its value $= x \times 100 + 0 \times 10 + y \times 1$
$= 100x + y$

Using the condition given in the problem statement, we get:

$$100x + y = 100x + 10y - \frac{1}{40}(100x + 10y)$$

Simplifying the equation, we get:

$40(100x+y) = 40(100x+10y) - 100x - 10y$

$\therefore 4000x + 40y = 4000x + 400y - 100x - 10y$

$\therefore 400y - 40y - 10y = 100x$

$\therefore 350y = 100x$

$\therefore x = \frac{35y}{10}$

We know that x & y are whole numbers such that $1 \leq x \leq 9$ & $0 \leq y \leq 9$

Substituting possible values of y in our equation, we get:

For $y=0$, $x = \frac{35 \times 0}{10} = 0 \rightarrow$ Not possible since x can't be 0

For $y=1$, $x = \frac{35 \times 1}{10} = 3.5 \rightarrow$ Not possible since x has to be a whole number

For $y=2$, $x = \frac{35 \times 2}{10} = 7 \rightarrow$ Possible

For $y \geq 3$, $x \geq \frac{35 \times 3}{10}$, i.e., $x \geq 10.5 \rightarrow$ Not possible since $1 \leq x \leq 9$

$\therefore x = 7$ & $y = 2$ is the only possible solution & hence, the original number is $\underline{\underline{720}}$

Verification

Original Number = 720 → is a 3-digit multiple of 10

Number obtained by interchanging units and tens digit = 702

Original number $- \frac{1}{40} \times$ Original Number

$= 720 - \frac{1}{\cancel{40}} \times \overset{18}{\cancel{720}} = 702 =$ Number obtained by interchanging units and tens digits.

Hence, verified.