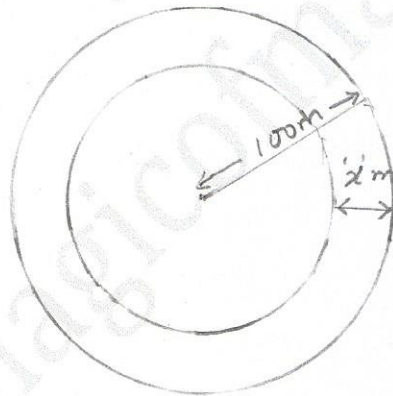


Problem

A circular pathway of uniform width needs to be built inside a circular garden such that the pathway touches the boundary of the garden throughout. Given that the garden has a radius of 100m and the area of the pathway is 19% of the area of the garden, find the width of the pathway. Also, find the ratio of the inner circumference of the pathway to its outer circumference.

Solution

The figure below shows 2 concentric circles:



The outer circle depicts the boundary of the garden as well as the outer boundary of the pathway.

The inner circle depicts the inner boundary of the pathway.

It is given that the radius of the outer circle = 100 metres

Let the width of the pathway be 'x' metres

Then, the radius of the inner circle = $(100-x)$ metres

$$\begin{aligned} \therefore \text{Area of garden} &= \text{Area of outer circle} = \pi \times (\text{radius of outer circle})^2 \\ &= \pi \times (100)^2 \text{ sq m} \end{aligned}$$

$$\begin{aligned} \& \text{ Area of pathway} &= \text{Area of outer circle} - \text{Area of inner circle} \\ &= \pi \times (\text{radius of outer circle})^2 \\ &\quad - \pi \times (\text{radius of inner circle})^2 \\ &= \pi \times (100)^2 - \pi \times (100-x)^2 \text{ sq m} \\ &= \pi \times [(100)^2 - (100-x)^2] \text{ sq m} \\ &= \pi (100+100-x)(100-100+x) \text{ sq m} \left[\begin{array}{l} \text{Using} \\ a^2-b^2 \\ = (a+b)(a-b) \end{array} \right] \\ &= \pi (200-x)x \text{ sq m} \end{aligned}$$

Given that area of pathway = 19% of area of garden, we have,

$$\pi (200-x)x = \frac{19}{100} \times \pi \times (100)^2$$

$$\therefore 200x - x^2 = 1900$$

$$\therefore x^2 - 200x + 1900 = 0$$

Splitting the middle term, we have

$$x^2 - 190x - 10x + 1900 = 0$$

$$\therefore x(x-190) - 10(x-190) = 0$$

$$\therefore (x-10)(x-190) = 0$$

$$\therefore x = 10 \text{ or } 190$$

x can't be 190, since the width of the pathway can't exceed the radius of the garden.

$$\therefore x = 10$$

$$\therefore \text{Width of pathway} = \underline{\underline{10 \text{ metres}}}$$

$$\begin{aligned}
 \text{Ratio of the inner circumference of pathway to its outer circumference} &= \frac{2\pi \times (\text{Radius of inner circle})}{2\pi \times (\text{Radius of outer circle})} \\
 &= \frac{(100-10)}{100} = \frac{90}{100} = \underline{\underline{9:10}}
 \end{aligned}$$

Verification

$$\text{Width of pathway} = 10 \text{ metres}$$

$$\text{Radius of outer circle} = 100 \text{ metres}$$

$$\therefore \text{Radius of inner circle} = (100-10) \text{ metres} = 90 \text{ metres}$$

$$\begin{aligned}
 \text{Area of pathway} &= \pi \times (100)^2 - \pi (90)^2 = \pi \times [10000 - 8100] \\
 &= \pi \times 1900 \text{ sq.m}
 \end{aligned}$$

$$19\% \text{ of area of garden} = \frac{19}{100} \times \pi \times (100)^2 = 1900\pi \text{ sq.m}$$

\therefore Area of pathway = 19% of area of garden & hence, verified.