Problem: A single complete graph needs to be constructed by taking apart the edges of three separate complete graphs having 3, 6 & 18 vertices respectively and assembling the edges together. Is this possible and if so, what is the order of the resultant graph?

Solution: Let K_n represent the complete graph of order n, i.e., K_n is a complete graph with n vertices.

We know that the no. of edges in $K_n = \frac{n(n-1)}{2}$

Using this formula, we can find the number of edges in the 3 given graphs

No. of edges in $K_3 = \frac{3(3-1)}{2} = \frac{3 \times 2}{2} = 3$

No. of edges in $K_6 = \frac{6(6-1)}{2} = \frac{6 \times 5}{2} = 15$

No. of edges in $K_{18} = \frac{18(18-1)}{2} = \frac{\overset{9}{\cancel{18}} \times 17}{\cancel{2}} = 153$

$\therefore$ Total no. of edges available $= 3 + 15 + 153$
$= 171$

Let us assume that we are able to construct a complete graph of order x (K_x) using the given edges

Then, no. of edges of $K_x = \dfrac{x(x-1)}{2} = 171$

If this equation has a +ve integer solution, then it is possible to construct a complete graph

Solving for x,

$$\frac{x^2 - x}{2} = 171$$

$$\Rightarrow x^2 - x = 2 \times 171 = 342$$

$$\Rightarrow x^2 - x - 342 = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1 + 4(342)}}{2} = \frac{1 \pm \sqrt{1369}}{2}$$

$$\Rightarrow x = \frac{1 \pm 37}{2} = \frac{38}{2}, \frac{-36}{2} = 19, -18$$

We can discard -18 as it is a -ve integer.

$\therefore$ 19 which is a +ve integer is the solution that we are looking for.

$\therefore$ It is possible to construct a complete graph with the given edges & the resultant graph has 13 vertices, i.e., the order of the resultant graph is 13.