

Problem: Show that  $A \cap B = A \cap C$  need not imply  $B = C$

Solution:

Let  $x \in B$  &  $x \notin A$

Then,  $x \notin A \cap B$  [By definition]

$\therefore x \notin A \cap C$  [ $\because A \cap B = A \cap C$ ] - ①

Also  $x \notin A$  [Assumed] - ②

① & ②  $\Rightarrow$  Either  $x \in C$  or  $x \notin C$  [By definition of intersection]

We saw that an element of  $B$  need not belong to  $C$ .

$\therefore B$  need not be equal to  $C$ .

Or in other words,

$A \cap B = A \cap C$  need not imply  $B = C$