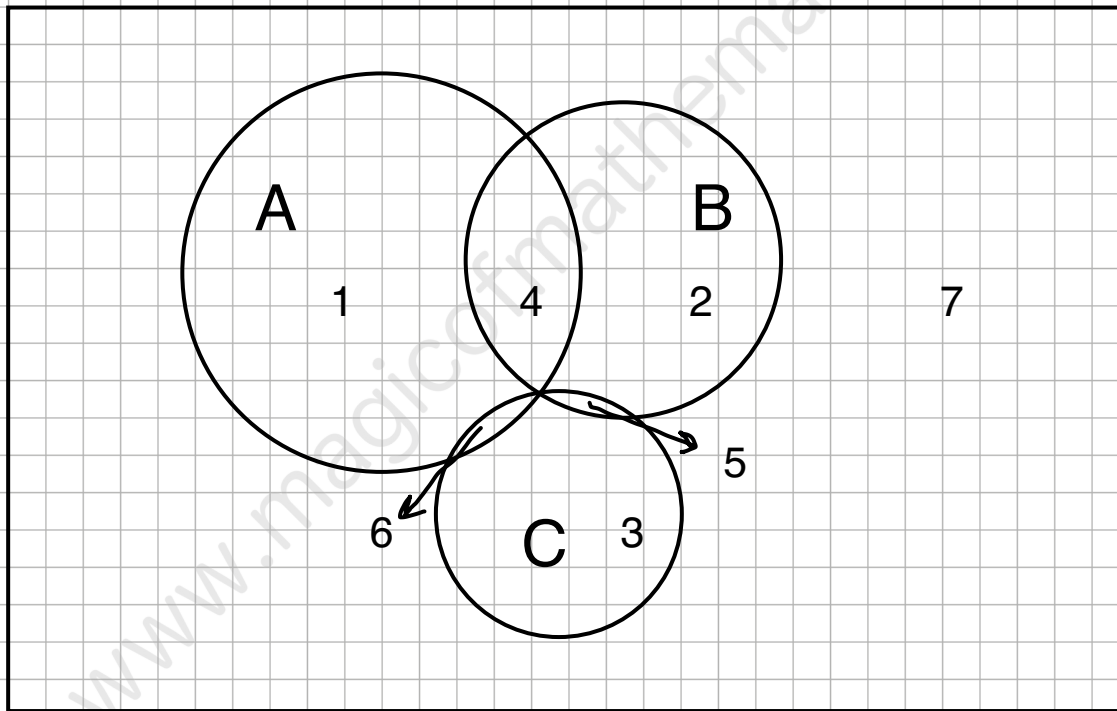


Problem: Find sets A, B & C such that
 $A \cap B, B \cap C$ & $C \cap A$ are non-empty sets &
 $A \cap B \cap C = \emptyset$

Solution: Let us construct a Venn diagram
to depict the conditions in the given
problem, as follows:



Region labeled 1, 2 & 3 depict elements
belonging only to A , only to B & only to C
respectively.

Regions 4, 5 & 6 represent $A \cap B$, $B \cap C$ & $C \cap A$ respectively.

Region 7 represents elements which do not belong to any of the three sets & is of no interest to us.

We should note that there is no region corresponding to $A \cap B \cap C$ as $A \cap B \cap C = \emptyset$ [Given]

Let us now construct possible sets A , B & C .

Let us assign elements a , b & c to regions 1, 2 & 3 respectively.

$$\therefore a \in A \text{ [but } a \notin B \text{ \& } a \notin C]$$

$$b \in B \text{ [but } b \notin A \text{ \& } b \notin C]$$

$$c \in C \text{ [but } c \notin A \text{ \& } c \notin B]$$

Let us now assign elements d , e & f to regions 4, 5 & 6 respectively.

$\therefore d \in A \cap B$ which means $d \in A$ & $d \in B$

Similarly, $e \in B$ & $e \in C$

And $f \in C$ & $f \in A$.

We can use all the above to construct possible sets A, B & C as

$$A = \{a, d, f\}$$

$$B = \{b, d, e\}$$

$$C = \{c, e, f\}$$

Verifying,

$$\left. \begin{aligned} A \cap B &= \{d\} \\ B \cap C &= \{e\} \\ C \cap A &= \{f\} \end{aligned} \right\} \rightarrow \text{Non empty}$$

$$\& A \cap B \cap C = \emptyset$$