**Problem**: $f: \mathbb{R} \to \mathbb{R}$ is a function defined by $f(x) = x^2$

(a) Is $f$ one-one? Why/Why not?

(b) Is $f$ onto? Why/Why not?

**Solution**:

(a) $f(-1) = (-1)^2 = 1$

$f(1) = 1^2 = 1$

$\therefore f(-1) = f(1) = 1$

$\therefore f$ is not one-one [$\because$ a one-one function can't have two elements of the domain mapping onto the same element]

(b) Let $y \in \mathbb{R}$ be an arbitrary element such that $f(x) = y$ for some $x$ from the domain of $f$

$\therefore \quad x^2 = y$

But $x^2 \geq 0$ for all $x \in \mathbb{R}$

$\therefore y \geq 0$

$\therefore$ For $y < 0$, there exists no $x$ from the domain of $f$ such that $f(x) = y$

$\therefore$ $f$ is not onto [$\because$ not every element of $Y$ is the image of some element of $X$]