Problem: Show that the function $f:\left[-\frac{1}{2},\infty\right) \to [-6,\infty)$ given by $f(x) = 4x^2+4x-5$ is invertible. Find the inverse of f.

Solution: For the function f to be invertible, it should be both one-one & onto. i.e., no two elements of the domain $\left[-\frac{1}{2},\infty\right)$ should map onto the same element of the co-domain $[-6,\infty)$ (one-one) & all the elements of the co-domain should have a pre-image in the domain under the function f (onto).

In other words, an arbitrary element belonging to the co-domain $[-6,\infty)$ should have one & only one pre-image in the domain $\left[-\frac{1}{2},\infty\right)$.

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Let us consider $y \in [-6, \infty)$ to be that arbitrary element. Let us see whether y has a unique pre-image in the domain $[-\frac{1}{2}, \infty)$.

If y has to have a unique pre-image in $[-\frac{1}{2}, \infty)$ under the function f, there should exist a unique $x \in [-\frac{1}{2}, \infty)$ such that $f(x) = y$

Since $f(x) = 4x^2 + 4x - 5$

$\therefore 4x^2 + 4x - 5 = y$

We are trying to establish the existence of x.

$\therefore$ We will try to express x in terms of y.

For that, we will use the technique of completing the square on the L.H.S

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Doing this, we get

$$(2x)^2 + 2(2x) + 1 - 5 - 1 = y$$

$$\Rightarrow (2x+1)^2 - 6 = y$$

$$\Rightarrow (2x+1)^2 = y+6$$

$$\Rightarrow (2x+1) = \pm\sqrt{y+6}$$

$$\Rightarrow x = \frac{\pm\sqrt{y+6} - 1}{2}$$

$$\therefore x = \frac{\sqrt{y+6} - 1}{2} \quad \text{or} \quad x = \frac{-\sqrt{y+6} - 1}{2}$$

Since $y \in [-6, \infty)$, $y \geq -6$

$$\therefore y+6 \geq 0$$

$$\therefore \sqrt{y+6} \geq 0 \quad \& \quad -\sqrt{y+6} \leq 0$$

$$\therefore \sqrt{y+6} - 1 \geq -1 \quad \& \quad -\sqrt{y+6} - 1 \leq -1$$

$$\therefore \frac{\sqrt{y+6} - 1}{2} \geq -\frac{1}{2} \quad \& \quad \frac{-\sqrt{y+6} - 1}{2} \leq -\frac{1}{2}$$

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Since $x \in [-\frac{1}{2}, \infty)$,

$x = \frac{-\sqrt{y+6} - 1}{2} \leq -\frac{1}{2}$ is not possible.

$\therefore x = \frac{\sqrt{y+6} - 1}{2}$ which is $\geq -\frac{1}{2}$

is the unique pre-image of the arbitrary element $y \in [-6, \infty)$

$\therefore f$ is both one-one & onto & hence invertible.

To find the inverse of f, we need to find a fn g such that

$fog = I_y$ & $gof = I_x$ in which case g will be the inverse of f.

Given a element $x \in [-\frac{1}{2}, \infty)$, $f(x) = 4x^2 + 4x - 5$ maps onto an element,

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Say $y \in [-6, \infty)$

$\& \; x = \dfrac{\sqrt{y+6} - 1}{2}$ gives us the pre-image of this y.

$\therefore$ Let $g : [-6, \infty) \to [-\frac{1}{2}, \infty)$ be a fn defined by $g(y) = \dfrac{\sqrt{y+6} - 1}{2}$

If we take $x \in [-\frac{1}{2}, \infty)$ & apply function f, we get an element, say $y \in [-6, \infty)$. If we take this y and apply function g on it, we get back the original x.

i.e $gof(x) = g(f(x)) = x$

$\therefore gof = I_X$

Similarly, we can show that $fog = I_Y$ & hence, g is the inverse of f.

But, let us establish this formally.

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