Problem: Find the implicit domain & range of the function f given by $f(x) = \frac{2x-3}{3x+2}$

Solution: To find the implicit domain, we need to find the values of x for which the function f will yield an image.

In other words, we need to find the values of x for which $f(x)$ is defined.

$f(x) = \frac{2x-3}{3x+2}$ is of the form $\frac{p}{q}$

which is defined when $q \neq 0$.

$\therefore$ $f(x)$ is defined when $3x+2 \neq 0$

i.e, when $x \neq -\frac{2}{3}$

$\therefore$ x can take on any real value except $-\frac{2}{3}$

$\therefore$ Domain of $f = \mathbb{R} - \left\{-\frac{2}{3}\right\}$

Range of f is the set of all elements having a pre-image in the domain under the function f.

To find the range,
let us consider an arbitrary element y belonging to the range of f.
Then, there exists an x (at least one) $\in$ domain of f such that

$$f(x) = y$$

Since $f(x) = \frac{2x-3}{3x+2}$

$$\frac{2x-3}{3x+2} = y$$

To find out what this f looks like, we will need to express x in terms of y. Let us try to do that.

$$2x-3 = (3x+2)y$$

$$\therefore \ 2x-3 = 3xy + 2y$$

$$\therefore \ 2x - 3xy = 2y+3$$

$$\therefore \ x(2-3y) = 2y+3$$

$$\therefore \ x = \frac{2y+3}{2-3y}$$

For x to exist, $\frac{3+2y}{2-3y}$ should be defined

which means $2-3y \neq 0$ & $\therefore$ $y \neq \frac{2}{3}$

$\therefore$ There exists no pre-image under f for $y = \frac{2}{3}$ & for all other $y \in \mathbb{R}$, there exists a pre-image under f.

$\therefore$ Range of f is $\mathbb{R} - \left\{\frac{2}{3}\right\}$