Problem: $f : \mathbb{N} \to \mathbb{R} - \{0\}$ is a function defined by $f(x) = \frac{1}{x}$

(a) Is f one-one? Why/Why not?

(b) Is f onto? Why/Why not?

Solution:

(a) Let us assume that the function f is not one-one. Then there exist two distinct elements of the domain $\mathbb{N}$ of the function f which map onto the same element under f.

i.e., $f(x_1) = f(x_2)$ for 2 distinct elements x_1 & $x_2 \in \mathbb{N}$

which means $\frac{1}{x_1} = \frac{1}{x_2}$

But this gives us $x_1 = x_2$

$\therefore$ x_1 & x_2 are not distinct.

$\therefore$ f is not one-one.

(b)

Consider the element $\frac{2}{3} \in R - \{0\}$

For f to be onto, there should be a $x \in N$ such $f(x) = \frac{2}{3}$

i.e, for some $x \in N$

$$\frac{1}{x} = \frac{2}{3}$$

which gives us $x = \frac{3}{2} \notin N$

$\therefore$ there exists no x such that $f(x) = \frac{2}{3}$

$\therefore$ f is not onto.