

Rationalizing the denominator

$$\textcircled{1} \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{3 \times 2}}{2} = \frac{\sqrt{6}}{2}$$

$$\textcircled{2} \frac{\sqrt{7}}{\sqrt{5} + \sqrt{2}} = \frac{\sqrt{7}}{(\sqrt{5} + \sqrt{2})} \times \frac{(\sqrt{5} - \sqrt{2})}{(\sqrt{5} - \sqrt{2})}$$

$$= \frac{\sqrt{7} \times \sqrt{5} - \sqrt{7} \times \sqrt{2}}{(\sqrt{5})^2 - (\sqrt{2})^2}$$

$$= \frac{\sqrt{7 \times 5} - \sqrt{7 \times 2}}{5 - 2}$$

$$= \frac{\sqrt{35} - \sqrt{14}}{3}$$

$$\textcircled{3} \frac{\sqrt{5}}{\sqrt{7} + \sqrt{3} - \sqrt{10}}$$

$$= \frac{\sqrt{5}}{[(\sqrt{7} + \sqrt{3}) - \sqrt{10}]} \times \frac{[(\sqrt{7} + \sqrt{3}) + \sqrt{10}]}{[(\sqrt{7} + \sqrt{3}) + \sqrt{10}]}$$

$$= \frac{\sqrt{5} \times \sqrt{7} + \sqrt{5} \times \sqrt{3} + \sqrt{5} \times \sqrt{10}}{(\sqrt{7} + \sqrt{3})^2 - (\sqrt{10})^2}$$

$$= \frac{\sqrt{5} \times \sqrt{7} + \sqrt{5} \times \sqrt{3} + \sqrt{5} \times \sqrt{10}}{(\sqrt{7})^2 + (\sqrt{3})^2 + 2 \times \sqrt{7} \times \sqrt{3} - 10}$$

$$= \frac{\sqrt{5} \times \sqrt{7} + \sqrt{5} \times \sqrt{3} + \sqrt{5} \times \sqrt{10}}{\cancel{7} + \cancel{3} + 2 \times \sqrt{7} \times \sqrt{3} - \cancel{10}}$$

$$= \frac{\sqrt{5} \times \sqrt{7} + \sqrt{5} \times \sqrt{3} + \sqrt{5} \times \sqrt{10}}{2 \times \sqrt{7} \times \sqrt{3}} \times \frac{\sqrt{7} \times \sqrt{3}}{\sqrt{7} \times \sqrt{3}}$$

$$= \frac{\sqrt{5} \times \sqrt{7} \times \sqrt{7} \times \sqrt{3} + \sqrt{5} \times \sqrt{3} \times \sqrt{7} \times \sqrt{3} + \sqrt{5} \times \sqrt{10} \times \sqrt{7} \times \sqrt{3}}{2 \times 7 \times 3}$$

$$= \frac{7\sqrt{15} + 3\sqrt{35} + \sqrt{5} \times \sqrt{5} \times \sqrt{2} \times \sqrt{7} \times \sqrt{3}}{42}$$

$$= \frac{7\sqrt{5} + 3\sqrt{35} + 5\sqrt{42}}{42}$$

$$\textcircled{4} \frac{\sqrt{3}}{\sqrt{5} + \sqrt{7} - \sqrt{2}} = \frac{\sqrt{3}}{(\sqrt{5} - \sqrt{2}) + \sqrt{7}}$$

$$= \frac{\sqrt{3}}{[(\sqrt{5} - \sqrt{2}) + \sqrt{7}]} \times \frac{[(\sqrt{5} - \sqrt{2}) - \sqrt{7}]}{[(\sqrt{5} - \sqrt{2}) - \sqrt{7}]}$$

$$= \frac{\sqrt{3} \times \sqrt{5} - \sqrt{3} \times \sqrt{2} - \sqrt{3} \times \sqrt{7}}{(\sqrt{5} - \sqrt{2})^2 - (\sqrt{7})^2}$$

$$= \frac{\sqrt{3} \times \sqrt{5} - \sqrt{3} \times \sqrt{2} - \sqrt{3} \times \sqrt{7}}{(\sqrt{5})^2 + (\sqrt{2})^2 - 2\sqrt{5}\sqrt{2} - 7}$$

$$= \frac{\sqrt{3} \times \sqrt{5} - \sqrt{3} \times \sqrt{2} - \sqrt{3} \times \sqrt{7}}{5 + 2 - 2\sqrt{5}\sqrt{2} - 7}$$

$$= \frac{\sqrt{3} \times \sqrt{5} - \sqrt{3} \times \sqrt{2} - \sqrt{3} \times \sqrt{7}}{-2\sqrt{5}\sqrt{2}} \times \frac{-\sqrt{5} \times \sqrt{2}}{-\sqrt{5} \times \sqrt{2}}$$

$$= -\sqrt{3} \times \sqrt{5} \times \sqrt{5} \times \sqrt{2} + \sqrt{3} \times \sqrt{2} \times \sqrt{5} \times \sqrt{2} + \sqrt{3} \times \sqrt{7} \times \sqrt{5} \times \sqrt{2}$$

$$2 \times 5 \times 2$$

$$= \frac{-5\sqrt{3} + 2\sqrt{15} + \sqrt{210}}{20}$$

$$\textcircled{5} \frac{\sqrt{2}}{\sqrt{5} - \sqrt{3} - \sqrt{2}} = \frac{\sqrt{2}}{\sqrt{5} - (\sqrt{3} + \sqrt{2})}$$

$$= \frac{\sqrt{2}}{[\sqrt{5} - (\sqrt{3} + \sqrt{2})]} \times \frac{[\sqrt{5} + (\sqrt{3} + \sqrt{2})]}{[\sqrt{5} + (\sqrt{3} + \sqrt{2})]}$$

$$= \frac{\sqrt{2} \times \sqrt{5} + \sqrt{2} \times \sqrt{3} + \sqrt{2} \times \sqrt{2}}{(\sqrt{5})^2 - (\sqrt{3} + \sqrt{2})^2}$$

$$= \frac{\sqrt{2} \times \sqrt{5} + \sqrt{2} \times \sqrt{3} + \sqrt{2} \times \sqrt{2}}{5 - [(\sqrt{3})^2 + (\sqrt{2})^2 + 2\sqrt{3}\sqrt{2}]}$$

$$= \frac{\sqrt{2} \times \sqrt{5} + \sqrt{2} \times \sqrt{3} + \sqrt{2} \times \sqrt{2}}{5 - [3 + 2 + 2\sqrt{3}\sqrt{2}]}$$

$$5 - [3 + 2 + 2\sqrt{3}\sqrt{2}]$$

$$= \frac{\sqrt{2} \times \sqrt{5} + \sqrt{2} \times \sqrt{3} + 2}{5 - 3 - 2 - 2\sqrt{3}\sqrt{2}}$$

$$~~5 - 3 - 2 - 2\sqrt{3}\sqrt{2}~~$$

$$= \frac{\sqrt{2} \times \sqrt{5} + \sqrt{2} \times \sqrt{3} + 2}{-2\sqrt{3}\sqrt{2}} \times \frac{(-\sqrt{3}\sqrt{2})}{(-\sqrt{3}\sqrt{2})}$$

$$= \frac{-\sqrt{2} \times \sqrt{5} \times \sqrt{3} \times \sqrt{2} - \sqrt{2} \times \sqrt{3} \times \sqrt{3} \times \sqrt{2} - 2\sqrt{3}\sqrt{2}}{2 \times 3 \times 2}$$

$$2 \times 3 \times 2$$

$$= \frac{-2 \times \sqrt{15} - 3 \times 2 - 2\sqrt{3}\sqrt{2}}{2 \times 3 \times 2}$$

$$= \frac{-\sqrt{15} - 3 - \sqrt{6}}{6}$$

$$\begin{aligned}
 \textcircled{6} \quad \frac{\sqrt{3}}{\sqrt{3} + \sqrt{7} - 2} &= \frac{\sqrt{3}}{(\sqrt{3} - 2) + \sqrt{7}} \\
 &= \frac{\sqrt{3}}{(\sqrt{3} - 2) + \sqrt{7}} \times \frac{[(\sqrt{3} - 2) - \sqrt{7}]}{[(\sqrt{3} - 2) - \sqrt{7}]} \\
 &= \frac{\sqrt{3} \times \sqrt{3} - 2 \times \sqrt{3} - \sqrt{3} \times \sqrt{7}}{(\sqrt{3} - 2)^2 - (\sqrt{7})^2} \\
 &= \frac{3 - 2\sqrt{3} - \sqrt{3} \times \sqrt{7}}{(\sqrt{3})^2 + (2)^2 - 2(\sqrt{3})(2) - 7} \\
 &= \frac{3 - 2\sqrt{3} - \sqrt{3} \times \sqrt{7}}{\cancel{3} + \cancel{4} - 4\sqrt{3} - \cancel{7}} \\
 &= \frac{3 - 2\sqrt{3} - \sqrt{3} \times \sqrt{7}}{-4\sqrt{3}} \cdot \frac{(-\sqrt{3})}{(-\sqrt{3})} \\
 &= \frac{-3\sqrt{3} + 2 \times 3 + 3 \times \sqrt{7}}{4 \times 3}
 \end{aligned}$$

$$= \frac{-\sqrt{3} + 2 + \sqrt{7}}{4}$$

$$\textcircled{7} \quad \frac{\sqrt{2}}{\sqrt{3} + \sqrt{6} + \sqrt{2} + \sqrt{5}}$$

$$= \frac{\sqrt{2}}{(\sqrt{3} + \sqrt{5}) + (\sqrt{6} + \sqrt{2})} \times \frac{(\sqrt{3} + \sqrt{5}) - (\sqrt{6} + \sqrt{2})}{(\sqrt{3} + \sqrt{5}) - (\sqrt{6} + \sqrt{2})}$$

$$= \frac{\sqrt{2} \times \sqrt{3} + \sqrt{2} \times \sqrt{5} - \sqrt{2} \times \sqrt{6} - \sqrt{2} \times \sqrt{2}}{(\sqrt{3} + \sqrt{5})^2 - (\sqrt{6} + \sqrt{2})^2}$$

$$= \frac{\sqrt{6} + \sqrt{10} - \sqrt{12} - 2}{(\sqrt{3})^2 + (\sqrt{5})^2 + 2(\sqrt{3})(\sqrt{5}) - [(\sqrt{6})^2 + (\sqrt{2})^2 + 2(\sqrt{6})(\sqrt{2})]}$$

$$= \frac{\sqrt{6} + \sqrt{10} - \sqrt{12} - 2}{3 + 5 + 2\sqrt{15} - 6 - 2 - 2\sqrt{12}}$$

$$= \frac{\sqrt{6} + \sqrt{10} - \sqrt{12} - 2}{2(\sqrt{15} - \sqrt{12})}$$

$$= \frac{\sqrt{6} + \sqrt{10} - \sqrt{12} - 2}{2(\sqrt{15} - \sqrt{12})}$$

$$= \frac{\sqrt{6} + \sqrt{10} - \sqrt{12} - 2}{2(\sqrt{15} - \sqrt{12})}$$

$$= \frac{\sqrt{6} + \sqrt{10} - \sqrt{12} - 2}{2(\sqrt{15} - \sqrt{12})} \times \frac{(\sqrt{15} + \sqrt{12})}{(\sqrt{15} + \sqrt{12})}$$

$$= \frac{\sqrt{6 \times 15} + \sqrt{6 \times 12} + \sqrt{10 \times 15} + \sqrt{10 \times 12} - \sqrt{12 \times 15} - \sqrt{12 \times 12} - 2\sqrt{15} - 2\sqrt{12}}{2[(\sqrt{15})^2 - (\sqrt{12})^2]}$$

$$= \frac{\sqrt{2 \times 3 \times 3 \times 5} + \sqrt{6 \times 6 \times 2} + \sqrt{2 \times 5 \times 3 \times 5} + \sqrt{2 \times 5 \times 2 \times 6} - \sqrt{2 \times 2 \times 3 \times 3 \times 5} - 12 - 2\sqrt{15} - 2\sqrt{12}}{2(15 - 12)}$$

$$= \frac{3\sqrt{10} + 6\sqrt{2} + 5\sqrt{6} + 2\sqrt{30} - 6\sqrt{5} - 12 - 2\sqrt{15} - 2\sqrt{12}}{6}$$